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Geotextiles and Geomembranes 25 (2007) 336–345
Geotextiles and Geomembranes
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Inflection point method for predicting settlement of PVD improved soft clay under embankments

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Received 8 June 2006; received in revised form 23 March 2007; accepted 4 April 2007
Available online 5 June 2007

Abstract

Prefabricated vertical drains (PVDs) are being used to accelerate the consolidation of subsoil for construction of high embankments on soft ground. The construction is carried out in stages and the height of each stage construction depends on gain in strength of soft subsoil and target factor of safety. The inflection point method for estimating the degree of consolidation for vertical drainage has previously been published. The degree of consolidation was estimated to be 70% at the inflection point. After 70% consolidation, the rate of consolidation reduces to a minimum value and it is economical and technically feasible to allow the second-stage loading. Additional load at this stage would ensure increase in shear strength of the subsoil. In this paper, the authors have extended the procedure of inflection point method for vertical drainage to a three-dimensional drainage when PVDs are adopted for subsoil improvement. Theoretical graphs have been developed which can estimate the percent consolidation at inflection point (% U_i) for different subsoil thickness, drain spacing ratios and C_r/C_v ratios. Different factors viz. smear, drain spacing, depth of clay thickness and well resistance were also considered in the analysis. The degree of consolidation at inflection point is observed in the range of 61–78%. The inflection point method has been applied to a settlement data from a case history of two sites and the estimated percent consolidation at inflection point has been compared with the values predicted from developed theoretical graphs. The total primary settlement estimated from inflection point has been compared with that of Asaoka method, which is widely used in the field. It is concluded that inflection point method has the potential for field application and provides an alternate method for estimating of total settlement in field applications using PVDs and surcharge, and to determine the appropriate required waiting period for stage loading.

Keywords: Inflection point; Degree of consolidation; Prefabricated vertical band drain (PVD); Soft clay; Settlement; Soil improvement

1. Introduction

The use of vertical drains and surcharge to accelerate the consolidation of thick deposits of soft clay for soil improvement is well established and there have been a number of recent papers of the topic (Shen et al., 2005; Chu et al., 2006; Abuel-Naga et al., 2006). The estimation of total primary settlement is very essential to arrive at actual degree of consolidation. The consolidation of subsoil is monitored by peizometers, settlement gauges, magnetic plates and inclinometers. The increase in subsoil strength is estimated by carrying out vane shear and static cone penetration tests before and after the surcharge loading. The theoretical curve between degree of consolidation (U_v) and time factor (T_v) obtained from the one-dimensional consolidation theory has been interpreted in different ways to estimate the degree of consolidation. Casagrande and Fadum (1940) used the theoretical curve between U_v and $\log T_v$ to estimate the time for 50% and 100% consolidation. The method devised by Taylor (1948) utilized the theoretical relationship between U_v and $\sqrt{T_v}$ for estimation of time for 90% consolidation. Sridharan and Rao (1981) estimated degree of consolidation, U_v by rectangular hyperbola fitting method. Pandian et al. (1992) estimated the degree of consolidation from the bilinear plot in the form of $\log(U_v/T_v)$ versus $\log T_v$. Cour (1971), Robinson (1997) and Mesri et al. (1999) have advocated the inflection point method for estimating the time for 70% consolidation.

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doi:10.1016/j.geotexmem.2007.04.004

Nomenclature

b	width of PVD
C_c	compression index
C_r	coefficient of consolidation in radial direction
C_u	undrained shear strength of subsoil
C_v	coefficient of consolidation in vertical direction
d	equivalent diameter of PVD/Geodrain
d_s	diameter of smear zone
D_e	effective drain spacing
e_0	initial void ratio
F	factor of safety
h	safe height of embankment over soft subsoil
H	thickness of clay layer
k_r	coefficient of permeability in radial direction
k'_r	coefficient of permeability in smear zone (radial direction)
k_v	coefficient of permeability in vertical direction
l	length of PVD
M_{field}	$dS_t/d\log t$ of field settlement curve
m_r	coefficient of volume change in radial direction
M_r	$dU/d\log T_r$
$M_{theoretical}$	$dU/d\log T_r$ of theoretical curve
M_v	$dU/d\log T_v$
n	drain spacing ratio $= D_e/d$
N_c	Terzaghi's bearing capacity factor
P_s	drain spacing for a square pattern
P_t	drain spacing for a triangular pattern
q_w	discharge capacity of PVD
R^2	coefficient of regression
s	smear ratio, d_s/d
S_t	settlement at time t
S_{t-1}	settlement at time $t-1$
S_T	total expected settlement in the field
S_{ult}	ultimate primary settlement
t	time
t_b	thickness of PVD
t_w	waiting period in days
T_r	time factor in radial direction
T_v	time factor in vertical direction
U	degree of Consolidation
U_i	degree of Consolidation at inflection point
U_r	degree of Consolidation in radial direction
U_v	degree of Consolidation in vertical direction
z	distance from the top boundary

Greek symbols

γ	density of subsoil
γ_d	dry density of embankment material
γ_w	density of water
μ	Poisson ratio of soil
$\Delta\sigma$	incremental over burden pressure
σ'_v	effective vertical stress
σ_0	effective over burden pressure

All these methods are based on one-dimensional consolidation without any consideration to radial or three-dimensional consolidation.

The significant aspect for embankment construction on soft soil is to reduce the time required for consolidation of soft ground. For this purpose, prefabricated vertical band drains (PVDs) are often used to accelerate the consolidation of soft soils (e.g. Hansbo, 1979, 2005; Holtz, 1987; Bergado et al., 1990, 1993a, b, 1996; Chai et al., 2001; Bo et al., 2003, 2005; Indraratna et al., 2005). The performance of vertical drain is typically evaluated using mathematical models. Analytical solutions developed by Barron (1948) and Hansbo (1981) are widely used. Bergado et al. (1991) studied smear effects due to installation of PVDs in the laboratory using special equipments and in the field using a full-scale test embankment. Li and Rowe (2001) examined the combined effect of vertical drains and geosynthetic reinforcement on the performance of embankment on soft clays. Chu et al. (2004) discussed different factors affecting the performance of vertical drains such as quality of drain, selection of soil parameters, smear effect and the type of mandrel used. Leo (2004) estimated the degree of consolidation after considering the well resistance and smear effect. Walker and Indaratana (2006) observed that permeability reduces parabolically towards vertical drain in the smear zone. Ladd (1991) and Bergado et al. (2002) studied and analyzed the stability and gain in strength of subsoil due to surcharge loading in multi stage construction. Nicholson and Jardine (1981) carried out multistage loading analysis based on the assumption that consolidated undrained strength (C_u) would grow in proportion to their maximum effective vertical stress (σ'_v). An overall ratio ($C_u/(\sigma'_v)$ of 0.25 was selected after considering the likely effects of change to plain strain condition, anistrophy and strain rate dependence. Ladd and Foot (1974) have given SHANSEP (stress history and normalized soil engineering properties) procedures for estimation of in situ undrained strength. Shibata and Sekiguchi (1981) tentatively suggested that load on embankment should be stopped when lateral deformability factor, i.e. ratio of load increment and horizontal incremental displacement reduces to around 200 kN/m^3.

The estimation of degree of consolidation or ultimate settlement of the soft clay is significant in multistage construction. This can be estimated by field instrumentation with the use of settlement gauges and piezometers. The field settlement can be analyzed by different methods to predict the ultimate settlement of the soft soil from the time of initial installation. Asaoka (1978) has suggested a procedure modified for application to consolidation problems with vertical drains using Barron (1948) solution for pure radial drainage. Tan (1994) proposed hyperbolic

fitting methods to estimate the total settlement due to consolidation with combined vertical and radial flow which occur when vertical drains are used to speed up consolidation. The application of both Asaoka and hyperbolic methods for the assessment of degree of consolidation of soft clays with PVDs in land reclamation projects have been described by Bo et al. (2003) and Arulrajah et al. (2004). The prediction of ultimate settlement by Asaoka method is affected by period of assessment after surcharge placement as well as by the time interval used for the assessment. At small intervals, it is difficult to assess the best-fit line through the data points. A larger time interval would require a long-term instrumentation monitoring program in order to assess the best-fit line through the data points. The prediction of ultimate settlement by hyperbolic method is affected by period of assessment after surcharge placement. In the inflection point method, as proposed by the authors, overcome these difficulties by plotting time on logarithmic scale and slope of U-log T_r on linear scale. The percent consolidation at inflection point (% U_i) is evaluated at different ratios of coefficient of radial drainage (C_r) and coefficient of vertical drainage (C_v). The ratio ranged between 1 and 3. The estimated degree of consolidation is compared with that evaluated from the field settlement data from two sites of a case history.

2. Inflection point method for vertical drainage

By plotting the Terzaghi's theoretical time–settlement curve in semi logarithmic coordinates U% versus log T_v, it is observed that the curve has an inflection point corresponding to $T_v = 0.405$ and $U_i = 70\%$. Cour (1971) located the inflection point for the real time curve and fixed the coordinate of inflection point either by visualization or where the absolute value of the tangent to the time curve reaches to the maximum as shown in Fig. 1. Robinson (1997) provided an alternative procedure for location of inflection point. The slope of the curve, i.e. $M_v = \mathrm{d}U/\mathrm{d}\log T_v$, is plotted against log T_v. In this, $\mathrm{d}U/\mathrm{d}\log T_v$ is the

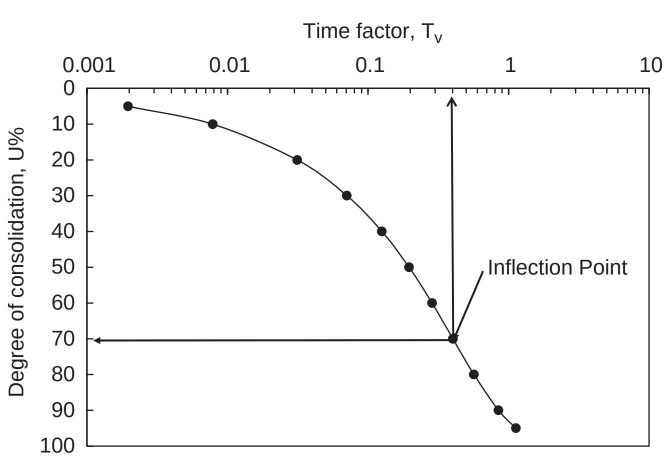

Fig. 1. Theoretical U–log T_v curve (after Cour, 1971).

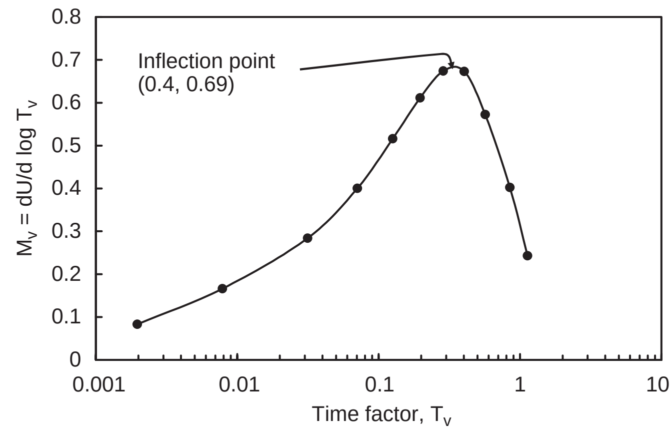

Fig. 2. M_v–log T_v curve showing inflection point (after Robinson, 1997).

ratio of change in degree of consolidation with corresponding change in log T_v. The observed maximum slope of the curve is considered as inflection point as shown in Fig. 2. Corresponding to this inflection point, the degree of consolidation is read from Fig. 1. The observed degree of consolidation is 70% similar to the value estimated by Cour (1971). But these methods are applicable to only one-dimensional vertical drainage and considering the wide applications of PVDs for soft soil improvement, authors have extended this theory to a three-dimensional drainage as discussed in Section 3.

3. Analytical approach of inflection point method in PVD-improved subsoil

The PVDs has been extensively used in the country, as some of the roads constructed under national highway development program (NHDP) run over difficult soft subsoils. In this program, the embankment is constructed in stages and observations of different instruments, viz. settlement gauges, piezometers and inclinometers are recorded over the construction period. For each stage loading, 90% degree of consolidation was achieved as per IS 15284 (Part 2, 2004). The height of each stage construction is evaluated based on improvement of shear strength of subsoil and to achieve a factor of safety of 1.25. But it is technically feasible and economical to restrict the waiting period for each stage loading to degree of consolidation at inflection point. The procedure of extension of inflection point method for a three-dimensional drainage as in PVDs application for subsoil improvement has been discussed below.

The consolidation process in soils of low permeability accelerated by vertical drains can be evaluated by Carrillo (1942) and Barron (1948) solution. According to Hansbo (1981), considering the effect of smear and well resistance factors on the combined degree of consolidation U, due to vertical drainage U_v and radial drainage U_r is given by

$$U = 1 - (1 - U_v)(1 - U_r). \quad (1)$$

The vertical degree of consolidation $U_v(t)$, is given by

$$U_v(t) = \sqrt{\frac{4 \times T_v}{\pi}}, \quad \text{as } U_v(t) < 60\% \tag{2}$$

(Atkinson and Eldred, 1981). The time factor (T_v) for vertical drainage is

$$T_v = \frac{C_v t}{H^2}, \tag{3}$$

where H is the thickness of clay layer, t is time, C_v is coefficient of consolidation in the vertical direction.

The radial degree of consolidation U_r is given by

$$U_r(t) = 1 - \exp\left[\frac{-8T_r}{\mu}\right]. \tag{4}$$

The time factor (T_r) for radial drainage is

$$T_r = \frac{C_r t}{D_e^2}. \tag{5}$$

D_e is the effective drain spacing, C_r is coefficient of consolidation in the radial direction

$$C_r = \frac{k'_r}{m_r}\gamma_w, \tag{6}$$

$$\mu = \ln n - \frac{3}{4} + \left(\frac{k_r}{k'_r} - 1\right)\ln s + \pi\frac{2l^2 k_r}{3q_w} = F_n + F_s + F_w, \tag{7}$$

where F_n is the drain spacing factor, $n = D_e/d$ is drain spacing ratio, D_e is $1.13P_s$ (drain spacing for a square pattern), D_e is $1.05P_t$ (drain spacing for a triangular pattern), F_s is smear factor where, s is d_s/d = smear ratio, d_s is diameter of smear zone, $d = 2(b + t_b)/\pi$ is equivalent diameter of PVD (Hansbo, 1981), t_b is thickness of PVD, k_r is coefficient of permeability in radial direction, k'_r is coefficient of permeability in smear zone (radial direction), m_r is coefficient of volume change in radial direction, F_w is well resistance factor, $q_w = 5k_r l^2$ (Bo, 2004), q_w is discharge capacity of PVD, l is length of PVD, $k'_r = k_v = 1 \times 10^{-9}$ m/s (Bergado et al., 1991; Chu et al., 2002), $k_r/k_v = 3$

Eqs. (1)–(7) could be used for determining the theoretical time–settlement curve for any practical problem where PVDs have been used for subsoil improvement. The degree of consolidation at inflection point depends on depth of subsoil H, the values of C_v and C_r, size and spacing of PVDs, drain influence diameter (D_e), and smear ratio which depends on pattern of laying of PVD.

Detailed theoretical analysis has been carried out to determine the variation of degree of consolidation at inflection point with drain spacing ratio ($n = D_e/d$) for different depths of soft soil. Analysis was carried out by assuming dimension of PVD as 100 mm × 4 mm, smear ratio ($s = 2$), well resistance factor and for different drain spacing. The analysis has been carried out for a square pattern of laying of PVD and different C_r/C_v ratios (1, 2 and 3). The degree of consolidation (U) was estimated from Eq. (1) for different time periods (t) and corresponding T_r values. The slope of the curve, i.e. $M_{\text{theoretical}} = \mathrm{d}U/\mathrm{dlog}\, T_r$, is then plotted against $\log T_r$. The maximum slope of this curve is the inflection point. The degree of consolidation corresponding to this inflection point is noted. Theoretical

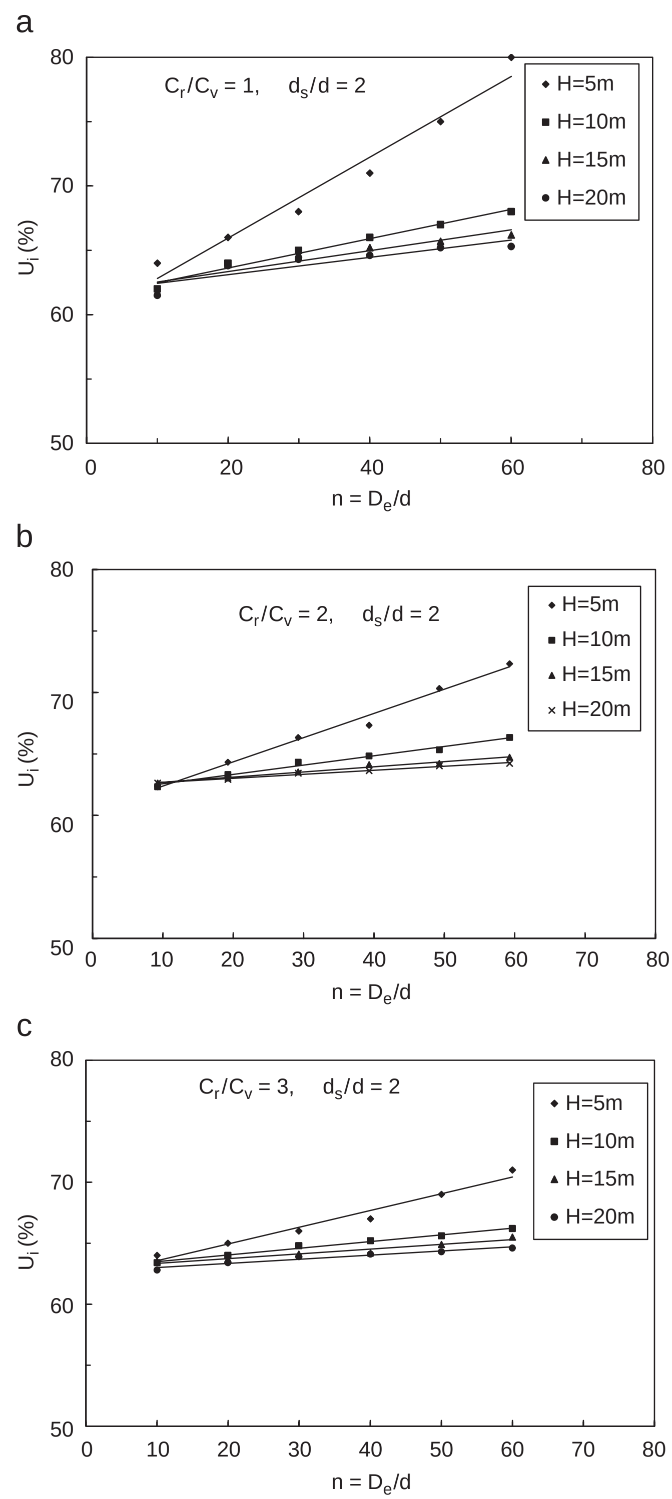

Fig. 3. (a) Variation of degree of consolidation at inflection point (% U_i) with drain spacing ratio (n) for different subsoil thickness and $C_r/C_v = 1$. (b) Variation of degree of consolidation at inflection point (% U_i) with drain spacing ratio (n) for different subsoil thickness and $C_r/C_v = 2$. (c) Variation of degree of consolidation at inflection point (% U_i) with drain spacing ratio (n) for different subsoil thickness and $C_r/C_v = 3$.

graphs have been developed which can estimate the degree of consolidation at inflection point (U_i) for different subsoil thickness, drain and soil parameters as shown in Figs. 3a–c. The applicability of these graphs was checked by determining the degree of consolidation at inflection point from the actual field settlement data as discussed in Section 4.

4. Practical approach of inflection point method to a case history

To authenticate the validity of the proposed method, data from two sites of a case history were collected. A total of 645 hectare of land was reclaimed for the construction of the Changi international airport in the Republic of Singapore in the year 1976–1978 (Tan, 1994). Different types of vertical drains viz. Sand drains and Geodrains were used to treat the underlying marine clays to minimize the differential settlement and to avoid costly maintenance of the airport runways, taxiways and parking aprons. Consolidation of subsoil was achieved by laying band drains in square pattern at different spacing. About 5 m surcharge was used for consolidation of soft subsoil. Data regarding band drain and soil characteristics for two sites is given in Table 1. Instrumentation was carried out to observe the subsoil performance. Magnetic settlement gauges were installed to record the time-dependent settlements.

Considering the drain dimensions and spacing, soil data and C_r/C_v ratio, Fig. 3c was used to estimate the theoretical degree of consolidation at inflection point. The theoretical values are observed to be 65.7% and 64.6%, respectively, for the two sites (Table 1). For the site 1, Geodrains are installed at 3.2 m spacing in square pattern and laid upto a depth of 20 m. The field time (t)–settlement (S_t) curve for the site 1 is shown in Fig. 4. The slope of the curve, i.e. $M_{\text{field}} = \mathrm{d}S_t/\mathrm{d}\log t$, is then plotted against $\log t$ as shown in Fig. 5. The time interval for plotting is taken as 10 days. The maximum slope of the best-fitted curve is the inflection point. The settlement corresponding to this inflection point was then noted from Fig. 4. The degree of consolidation is then estimated by the ratio of this value and the total settlement estimated from Eq. (11). Similar type of analysis made for the site 2. For the site 2, Geodrains were installed at 2.1 m spacing in square pattern upto a depth of 25 m at the same area of Changi airport as site 1. The time–settlement curve and the inflection point for the same is shown in Figs. 6 and 7, respectively. Table 2 compares the degree of consolidation at inflection point (% U_i) determined theoretically with that determined from the actual field settlement data. The degree of consolidation calculated to be 68% and 64%, respectively, for sites 1 and 2. It is observed that the values determined by theoretical

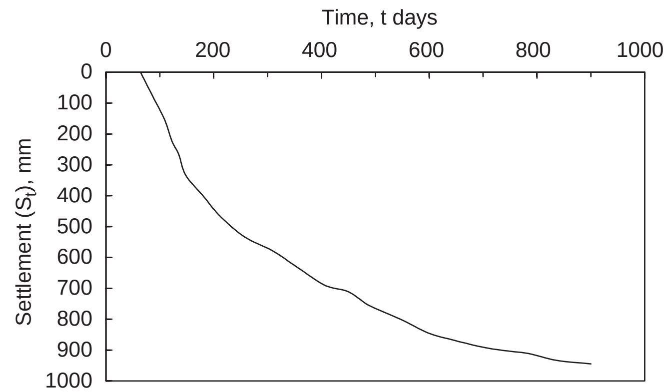

Fig. 4. Time–Settlement curve for site 1.

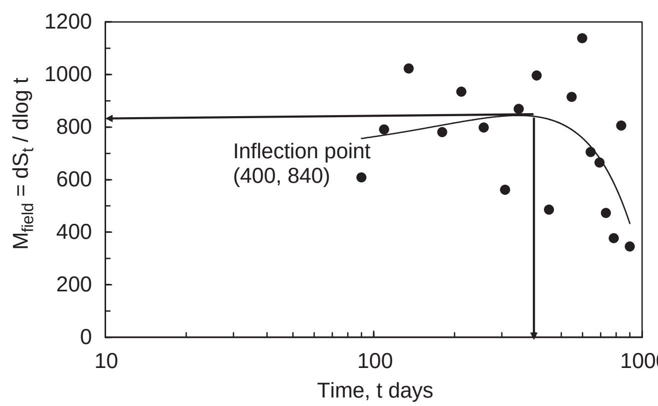

Fig. 5. M_{field}–$\log t$ curve for site 1.

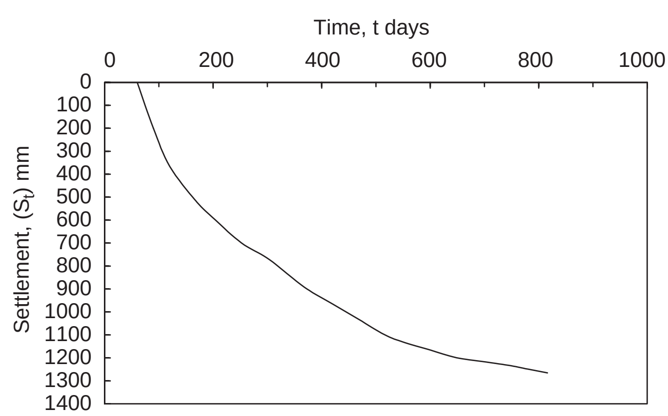

Fig. 6. Time–settlement curve for site 2.

Table 1
Predicted percent consolidation at inflection point for different drain and soil parameters

Site	Drain diameter d (m)	Drain spacing P_s (m)	$D_e = 1.13P_s$	$n = D_e/d$	H (m)	C_v (m^2/year)	C_r (m^2/year)	U_i (%)
1	0.065	3.2	3.616	55.6	10	1	3	65.7
2	0.065	2.1	2.373	36.5	12.5	1	3	64.6

U_i (%): Percent consolidation at inflection point.

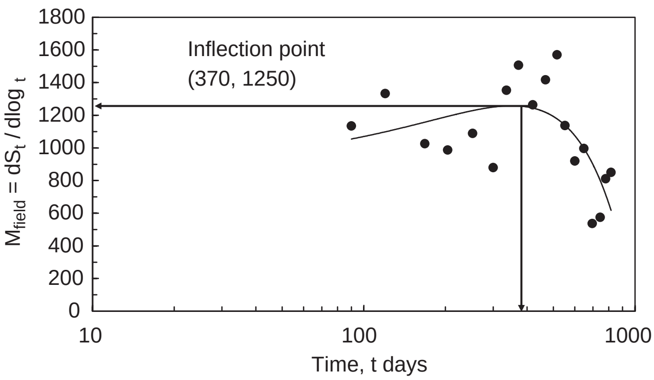

Fig. 7. M_{field}–$\log t$ curve for site 2.

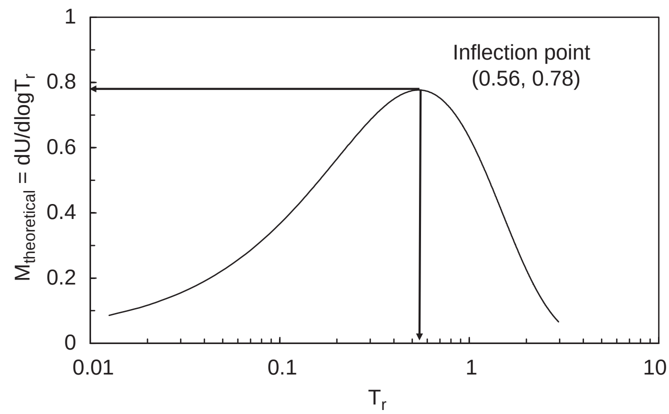

Fig. 8. Inflection point from $M_{\text{theoretical}}$–T_r curve for site 1.

Table 2
Comparison of predicted and actual percent consolidation at inflection point

Site	Total settlement from C_c (m)	Settlement at inflection point (m)	Percent consolidation at inflection point	
			Predicted from theoretical curve	Actual from field
1	1.02	0.69	65.7	68
2	1.45	0.92	64.6	64

C_c = Compression index parameter.

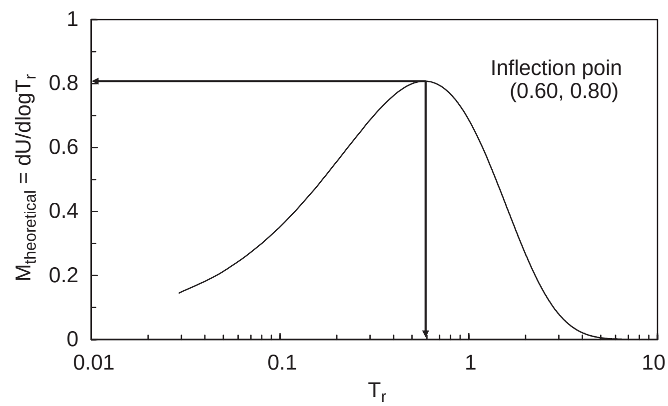

Fig. 9. Inflection point from $M_{\text{theoretical}}$–T_r curve for site 2.

and practical approach are comparable. However, it is observed that due to wide scatter in the settlement data, the best-fitted second-order polynomial curve has regression coefficients (R^2) of 0.33 and 0.51, respectively, for sites 1 and 2, respectively. This implies that the method of inflection point needs to be applied to settlement data from more sites to authenticate the values obtained from theoretical approach.

The inflection point method was also used to estimate the total settlement and compared with that estimated from Asaoka method, widely used for prediction of ultimate settlement. This has been discussed in Section 5 and Section 6.

5. Estimation of total primary settlement by inflection point method

Considering the field parameters for the two sites viz. dimension of band drain, drain spacing, depth of clay layers, C_r/C_v ratios, smear ratio and pattern of laying of PVD, the slope of the curve i.e. $M_{\text{theoretical}} = \mathrm{d}U/\mathrm{d}\log T_r$, is plotted against $\log T_r$ as discussed in Section 3. $M_{\text{theoretical}}$ is noted from the inflection point as shown in Figs. 8 and 9. The estimated values of M_{field} for the two sites (as discussed in Section 4) were then used to estimate the total primary settlement S_T from Eq. (2).

$$S_T = \frac{M_{\text{field}}}{M_{\text{theoretical}}}, \tag{8}$$

where M_{field} is the maximum slope at inflection point of $\mathrm{d}S_t/\mathrm{d}\log t - \log t$ curve (Figs. 5 and 7), $M_{\text{theoretical}}$ is maximum slope at inflection point of $\mathrm{d}U/\mathrm{d}\log T_r - \log T_r$ curve (Figs. 8 and 9)

$$S_T = \frac{840}{0.78} = 1077\,\text{mm} = 1.077\,\text{m} \quad \text{(site 1)},$$

$$S_T = \frac{1250}{0.80} = 1563\,\text{m} = 1.563\,\text{m} \quad \text{(site 2)}.$$

6. Comparison of inflection point method and Asaoka method

Asaoka (1978) method, a graphical method, which is at present widely used for prediction of total settlement. The method is applied to the field time–settlement data of the two sites for prediction of total settlement. Procedure for estimation of total settlement by Asaoka method for the two sites is shown in Figs. 10 and 11, respectively. The values obtained from the two methods have been compared as shown in Table 3. The total settlement estimated from inflection point method varies in the range of +5%

to +8% of total settlement estimated from Eq. (11). The variation of the same from Asoaka method was in the range of −8% to −10%.

7. Stepwise procedure for application of inflection point method

The inflection point method can be applied to a specific field problem. The method is very useful for estimating the degree of consolidation at inflection point and corresponding waiting period for each stage loading. The total expected primary settlement in the field can also be estimated. The stepwise procedure for the same is given below.

(1) Given the values of: Thickness of subsoil clay layer (H), Equivalent diameter of PVD (d), Effective drain spacing (D_e), C_r/C_v ratio, Smear ratio(s), Well resistance and drain spacing factor, the percent degree of consolidation at inflection point (% U_i) is estimated using Figs. 3a–c.

(2) For a given parameter in step 1, draw the curve $M_{theoretical}$ versus $\log T_r$ on a semilogarithamic sheet. The maximum slope of this curve corresponds to inflection point. The time factor T_r and maximum slope $M_{theoretical}$ corresponding to this inflection point is noted.

(3) The waiting period t_w (for % U_i as determined in step 1) is then predicted corresponding to the estimated value of T_r (step 2) and drain influence diameter, D_e using the formula

$$t_w = \frac{(D_e)^2 T_r}{C_r}. \tag{9}$$

(4) From the instrumented section, the observed time (t)–settlement (S_t) curve is plotted. The slope of the curve, i.e. $M_{field} = dS_t/d\log t$, is then plotted against $\log t$. The value of M_{field} corresponding to inflection point is then noted.

(5) From the values of $M_{theoretical}$ (step 2) and M_{field} (step 4), the total expected primary settlement in the field is estimated from the following equation:

$$S_T = \frac{M_{field}}{M_{theoretical}}. \tag{10}$$

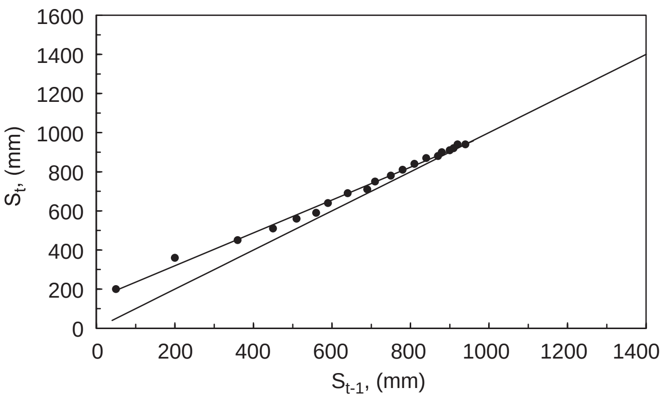

Fig. 10. Application of Asaoka method for settlement data from site 1.

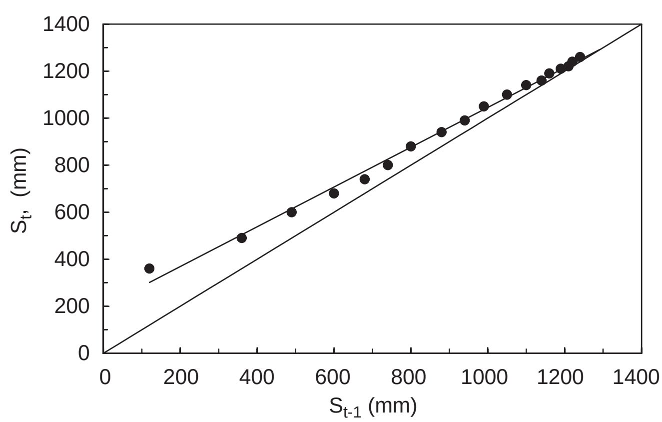

Fig. 11. Application of Asaoka method for settlement data from site 2.

8. Solved example

An embankment of 5 m height is to be constructed on a soft ground of clay layer thickness equal to 10 m. The undrained strength of subsoil is $C_u = 15\,kN/m^2$, compression index $C_c = 0.8$ and initial void ratio $e_0 = 2.1$. The dry density (γ_d) of the embankment material is $18\,kN/m^3$. The details of subsoil characteristics of the study are given in Fig. 12. Before construction of embankment, the subsoil strata is treated with a band drain. The width (b) and

Table 3
Comparison of total settlement estimated from Asaoka and Inflection point method

Site	Total primary settlement from C_c (m)	Predicted total field settlement (m)		Percent increase or decrease with respect to that evaluated from C_C	
		Asaoka method	Inflection point method	Asaoka method	Inflection point method
1	1.02	0.94	1.07	(−)8	(+)5
2	1.45	1.30	1.56	(−)10	(+)8

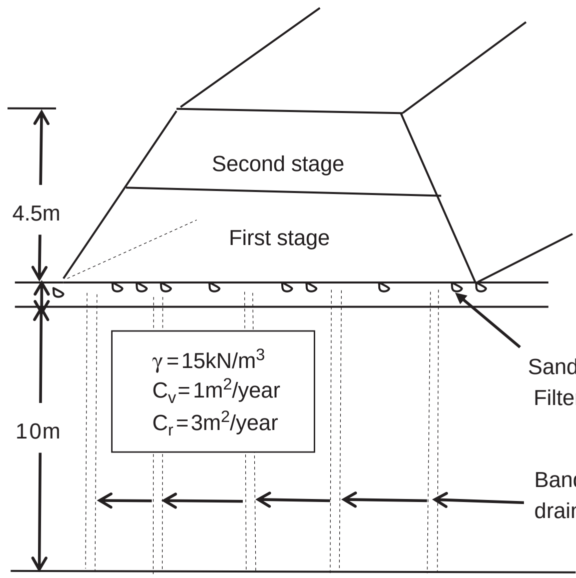

Fig. 12. Cross-sectional view of embankment for solved example.

thickness (t_b) of band drain is 0.1 m and 0.004 m, respectively. The spacing of band drain is 2 m center to center in the square pattern. The water table is at the top of ground surface.

Determine the degree of consolidation at inflection point (% U_i) and corresponding waiting period (t_w) and total expected primary settlement in the field.

Solution:

Safe height of embankment over soft subsoil h, given by

$$h=\frac{C_u N_c}{\gamma F}=\frac{15\times 5.7}{18\times 1.25}=3.8\,\mathrm{m},$$

Assuming the factor of safety (F) = 1.25.

So the embankment may be constructed upto a height of 3.8 m, as first stage loading.

Step 1: Equivalent diameter of band drain, $d=2(b+t_b)/\pi=0.066\,\mathrm{m}$, effective drain spacing $=D_e=1.13\times 2=2.26\,\mathrm{m}$, drain spacing ratio $=n=D_e/d=2.26/0.066=34.24$, from the given values, $H=10\,\mathrm{m}$, n, $C_r/C_v=3$, the degree of consolidation at inflection point is estimated from Fig. 3c as $U_i=65.1\%$.

Step 2: Based on $H=10\,\mathrm{m}$, $C_r/C_v=3$, and $n=34.24$, $M_{\mathrm{theoretical}}$ versus T_r is plotted as shown in Fig. 13. The inflection point corresponds to coordinate (0.50, 0.80). The radial time factor T_r corresponding to the inflection point is thus estimated as 0.50.

Step 3: Based on, $D_e=2.26\,\mathrm{m}$, $T_r=0.50$ and $C_r=3\,\mathrm{m}^2/\mathrm{year}$ and using the equation

$$T_r=\frac{C_r t_w}{(D_e)^2}.$$

Waiting periods (t_w) for first stage construction is $t_w=T_r(D_e)^2/C_r=0.50\times(2.26)^2/3=0.85$ years $=310$ days

Step 4: Settlement at inflection point after first stage loading

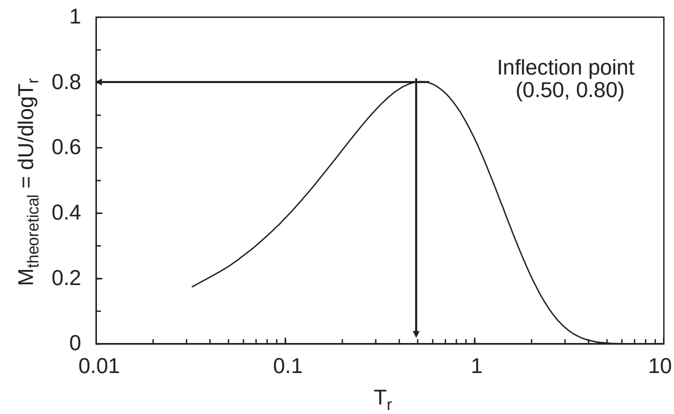

Fig. 13. Inflection point from $M_{\mathrm{theoretical}}$–T_r curve for solved example.

Ultimate primary settlement after first stage loading of 3.8 m.

$$S=\frac{H}{1+e_0}C_c\log_{10}\left(\frac{\sigma_0+\Delta\sigma}{\sigma_0}\right) \qquad (11)$$
$$=\frac{10}{1+2.1}\times 0.8\log_{10}\left(\frac{25.95+68.4}{25.95}\right)$$
$$=1.447\,\mathrm{m}.$$

Settlement at inflection point is $1.447\times 0.651=0.942\,\mathrm{m}$.

Net height of the embankment raised is 3.8–0.942 = 2.858 m.

Step 5: Height of embankment for second-stage loading

The first stage embankment construction stress is $3.8\times 18=68.4\,\mathrm{kN/m^2}$. Therefore, gain in undrained strength, C_u at 90% consolidation $=0.25\times 3.8\times 18=17.13\,\mathrm{kN/m^2}$ (Nicholson and Jardine, 1981)

Corresponding gain in strength, at 65.1% degree of consolidation is $=17.13\times 0.651=11.15\,\mathrm{kN/m^2}$.

Total undrained strength of subsoil $=15+11.15=26.15\,\mathrm{kN/m^2}$.

Safe height for the second-stage embankment construction is equal to

$$h=\frac{C_u N_c}{\gamma F}=\frac{26.15\times 5.7}{18\times 1.25}=6.62\,\mathrm{m}.$$

The height of second-stage loading could be arrived by trial and error process as given below:

(1) Iteration 1

Let the second-stage loading of embankment is say 2.5 m for first iteration. Total height of combined first- and second-stage loading is $2.858+2.5=5.358\,\mathrm{m}<6.62\,\mathrm{m}$. The ultimate total primary settlement for first-stage and second-stage construction (total height = 5.358 m) using Eq. (11) is 1.738 m. Net height of embankment $=5.358-1.738+0.942=4.562\,\mathrm{m}<5\,\mathrm{m}$ (required height), hence safe.

(2) Iteration 2

Now second-stage loading of embankment is say 3 m for second iteration. Total height of combined first- and

second-stage loading is $2.858+3=5.858\,m<6.62\,m$. The ultimate total primary settlement for first-stage and second-stage construction (total height = 5.858 m) using Eq. (11) is 1.817 m. Net height of embankment = 5.858–1.817 + 0.942 = 4.98 m < 5 m (required height), hence safe.

The iteration process would give the exact height of second-stage loading to be 3.02 m to achieve the total height of embankment to be 5 m. As there would be no significant change in C_r/C_v ratio of the subsoil after the second-stage loading, waiting period for the second-stage loading may also be kept as 310 days.

9. Conclusions

The inflection point method developed by Cour (1971) for vertical drainage has been extended to estimate the degree of consolidation at inflection point for a three-dimensional drainage where PVDs are adopted for subsoil improvement. The estimated value has been compared with the degree of consolidation at inflection point evaluated from actual field settlement data of a case history at two different sites. The expected total primary settlement from the inflection point method has been compared with that predicted from Asaoka method. Field application of inflection point method has also been discussed. Conclusions are summarized as follows;

(1) Theoretical graphs have been prepared which can estimate the percent consolidation at inflection point (% U_i) for different thicknesses of subsoil, specific size of PVD, different C_r/C_v ratios, considering smear, well resistance and drain spacing factors. The percent degree of consolidation at inflection point for 5 m thick depth of soft subsoil is estimated to be in the range of 62–78% whereas for thicknesses in the range of 10–20 m the values ranged between 61% and 66%.
(2) The percent degree of consolidation at inflection point method (% U_i) estimated from theoretical graphs for a specific field problem was observed to be comparable to that estimated from the actual field settlement data. But the method needs to be approved to settlement data for more sites to authenticate the values obtained from the theoretical approach.
(3) The total expected primary settlement determined from inflection point method over estimates that obtained from compressibility characteristics by 5–8%. The Asaoka method underestimates this value by 8–10%.
(4) The method is quite useful during stage construction of embankment on thick deposits of soft soil. The waiting period for each stage corresponding to degree of consolidation at inflection point (% U_i) can be estimated. The method provides an alternative to existing methods for determination of degree of consolidation of instrumented sections.

Acknowledgment

The kind approval of Director, CRRI, New Delhi, India 110020, to publish the research paper is acknowledged.

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